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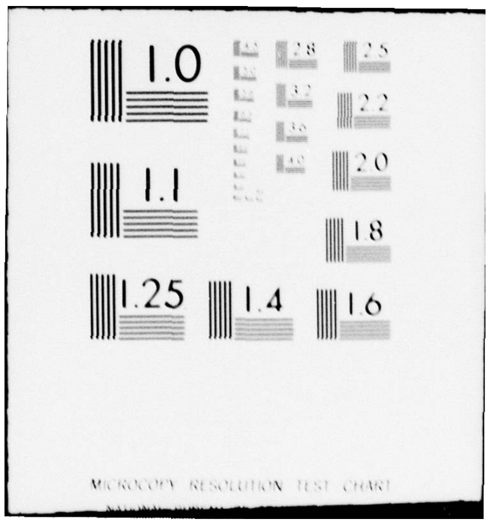
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Sound Propagation in the Sea:
ANALYTIC VERIFICATION OF WAVEFRONT SPREADING FORMULATIONS, NON-RECIPROCITY IN RAY REVERSAL, AND FNWF COMPUTER PROGRAM ,

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1. Definitions and Formulations

In Figure 1 we depict a segment of wavefront, $A\tau B$, of sound propagating in a vertical plane in the sea, having originated at the source S. The trace of a point τ on the wavefront defines a sound ray, $S\tau$, whose propagation instantaneously is along the wavefront-normal unit-vector $\mathbb{t}$ which makes the angle γ with the horizontal. This arbitrary ray left the source with angle γ_s.

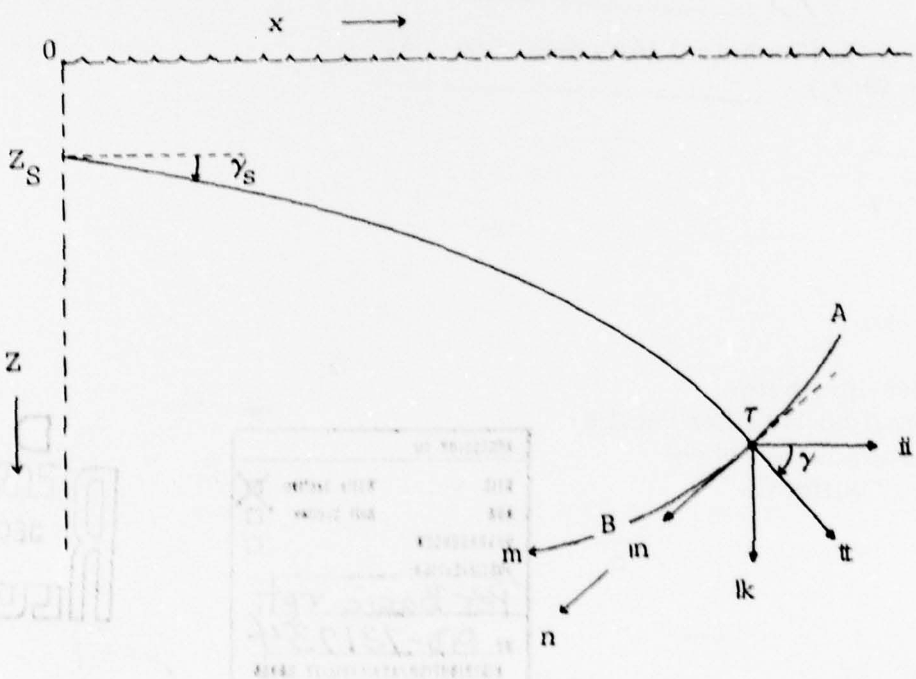

Fig. 1 Definitions

The unit vector $\mathbb{m}$ is wavefront tangent, clockwise normal to $\mathbb{t}$. Linear coordinates along the ray, $S\tau$, along the wavefront, τB, and along the <u>straight</u> line defined by the direction $\mathbb{m}$ at τ, are denoted by s, m and n respectively.

The formula for integrating the trace may be expressed by the equation

$$\mathbb{t} \cdot \nabla \gamma = - \frac{\cos \gamma}{R} - \frac{1}{C} \mathbb{m} \cdot \nabla C \tag{1}$$

The first term on the right hand side is a coordinate curvature term in which R is the local radius of curvature of the horizontal coordinate. The second term is the refraction where C is the non-uniform speed of sound in the sea.

Not only is the depth of the sea small compared to the radius of the earth, but it is also small compared to the earth's oblateness. This is why R is held constant, as the mean radius of the earth, in numerically tracing rays at the Fleet Numerical Weather Facility. Thus curvature of the horizontal coordinate is taken into account, but not strictly: it is given a constant value.

It should also be noted that the straight line defined by $\mathbb{m}$, and dashed in Fig. 1, takes on curvature when the sea surface is depicted as a straight line.

The formula for integrating the wavefront divergence along the trace may be obtained* by combining the spreading rate

$$\mathbb{t} \cdot \nabla L = L \, \mathbb{m} \cdot \nabla \gamma \tag{2}$$

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*"The wavefront-divergence factor in ray-intensity integration", M. M. Holl, Meteorology International Inc., Project M-140 Technical Note Two, Contract No. N62271-67-M-2000, April 1967.

with the refraction rate

$$\mathbb{t} \cdot \nabla \gamma = - \frac{1}{C} \mathbb{n} \cdot \nabla C \tag{3}$$

to obtain

$$\mathbb{t} \cdot \nabla (\mathbb{t} \cdot \nabla L) = - \frac{L}{C} \mathbb{n} \cdot \nabla (\mathbb{n} \cdot \nabla C) \tag{4}$$

L denotes the specific wavefront length per unit radian of emission angle, γ_s. Equation (4) may also be expressed in scalar form by

$$\frac{\partial^2 L}{\partial s^2} = - \frac{L}{C} \frac{\partial^2 C}{\partial m^2} \tag{5}$$

and by

$$\frac{\partial^2 L}{\partial s^2} = - \frac{L}{C} \left\{ \frac{\partial^2 C}{\partial n^2} - \frac{1}{L} \frac{\partial C}{\partial s} \frac{\partial L}{\partial s} \right\} \tag{6}$$

In treating the horizontal as straight, in applying Eq. (6), a curvature, relevant to, but negligible in, the term $\partial^2 C/\partial n^2$, is induced into the straight line defined by $\mathbb{n}$. However this curvature is not that which is consistent with the treatment adopted by Eq. (1). For analytical purposes consistency requires that we rederive the wavefront divergence equation by substituting Eq. (1) for Eq. (3) and subsequently neglecting the curvature of the horizontal.

The operation $\mathbb{t} \cdot \nabla$ on Eq. (2) yields

$$\mathbb{t} \cdot \nabla (\mathbb{t} \cdot \nabla L) =$$

$$(\mathbb{t} \cdot \nabla L)(\mathbb{n} \cdot \nabla \gamma) + L(\mathbb{t} \cdot \nabla \mathbb{n}) \cdot \nabla \gamma + L\, \mathbb{n} \cdot \nabla (\mathbb{t} \cdot \nabla \gamma) - L(\mathbb{n} \cdot \nabla \mathbb{t}) \cdot \nabla \gamma \tag{7}$$

We substitute

$$\mathbb{t} \cdot \nabla \mathbb{n} = - \mathbb{t} \;\; \mathbb{t} \cdot \nabla \gamma \tag{8}$$

$$\mathbb{n} \cdot \nabla \mathbb{t} = \mathbb{n} \;\; \mathbb{n} \cdot \nabla \gamma \tag{9}$$

and obtain

$$\mathbb{t} \cdot \nabla (\mathbb{t} \cdot \nabla L) =$$

$$(\mathbb{t} \cdot \nabla L)(\mathbb{n} \cdot \nabla \gamma) - L(\mathbb{t} \cdot \nabla \gamma)^2 + L\, \mathbb{n} \cdot \nabla (\mathbb{t} \cdot \nabla \gamma) - L(\mathbb{n} \cdot \nabla \gamma)^2 \tag{10}$$

We next selectively substitute from Eqs. (1) and (2):

$$\mathbb{t} \cdot \nabla (\mathbb{t} \cdot \nabla L) =$$

$$\cancel{L(\mathbb{n} \cdot \nabla \gamma)^2} - L\left\{\frac{\cos \gamma}{R} + \frac{1}{C}\, \mathbb{n} \cdot \nabla C\right\}^2 - L\, \mathbb{n} \cdot \nabla \left\{\frac{\cos \gamma}{R} + \frac{1}{C}\, \mathbb{n} \cdot \nabla C\right\}$$

$$- \cancel{L(\mathbb{n} \cdot \nabla \gamma)^2} \tag{11}$$

which may be developed into

$$\mathbb{t} \cdot \nabla (\mathbb{t} \cdot \nabla L) =$$

$$-\frac{L}{C}\, \mathbb{n} \cdot \nabla (\mathbb{n} \cdot \nabla C) - L\left\{\frac{2 \cos \gamma}{R}\, \frac{\mathbb{n} \cdot \nabla C}{C} + \frac{\cos^2 \gamma}{R^2} - \frac{\sin \gamma}{R}\, \frac{\mathbb{t} \cdot \nabla L}{L}\right\} \tag{12}$$

This version is consistent with tracing the ray according to Eq. (1). If we neglect the curvature altogether by setting $R = \infty$ then Eq. (12) reduces to Eq. (4) just as Eq. (1) reduces to Eq. (3).

Integration of L along a ray, by Eq. (12) or (6), from some point, requires specification of L and $\partial L/\partial s$ in order to proceed from that point. In beginning at the source, we specify

$$L_S = 0 \tag{13}$$

which is consistent with representing the source effectively as a point source. We also specify

$$(\partial L/\partial s)_S = 1 \tag{14}$$

for normalization of L as the specific wavefront length per unit radian of emission angle, γ_S.

In the next section we demonstrate by analytic example that the wavefront divergence generally depends not only on the path of the ray between source and receiver but also on which end of this path is the point source. Generally, the relative intensity loss changes in interchanging the position of the source and receiver.

Another way of looking at this fact is by the statement that the proportional change in intensity, in propagating through a finite segment of path in one direction, depends on the curvature of the wavefront at the commencement of the path segment. The integration is reversible if we duplicate the direct exit curvature of the wavefront for initializing the reverse traverse of the segment.

2. An Analytic Solution

In the present section we design a parameterized analytic solution, showing its complete derivation from Eq. (1). This solution serves to (a) test any of our derived equations, (b) edify aspects of the propagation, and (c) test and evaluate computer programs of the Fleet Numerical Weather Facility for numerical integration of Eqs. (1) and (6).

For speed-of-sound distributions which are functions of depth, Z, only, Eq. (1) reduces to

$$\frac{\partial \gamma}{\partial x} = -\frac{1}{R} - \frac{\partial K}{\partial Z} \tag{15}$$

where $K \equiv \ln C$. Our solution is for the special profile

$$C = C_0 \, e^{-kZ} \tag{16}$$

and a point source at arbitrary depth Z_S. The horizontal (i.e. range) coordinate is $x \geq 0$. We shall only solve for direct rays, not going into sea-surface or bottom reflections. The constant k may be positive or negative; we consider it to be positive for discussion purposes.

The Eq. (16) profile submitted to Eq. (15) yields

$$\frac{\partial \gamma}{\partial x} = \mu \tag{17}$$

where $\mu \equiv k - 1/R$ is considered to be a positive constant. Integration of Eq. (17) yields

$$\gamma = \gamma_S + \mu x \tag{18}$$

for any and all rays. All rays are refracted downwards, with maximum range

$$x_m = \frac{1}{\mu} (\pi/2 - \gamma_S) \tag{19}$$

as asymptote. The angle at which a ray leaves the point source is, by definition, limited to the range

$$-\pi/2 \leq \gamma_S \leq \pi/2 \tag{20}$$

For a ray defined by γ_S we have

$$\frac{dZ}{dx} = \tan \gamma = \tan (\gamma_S + \mu x) \tag{21}$$

Integration yields

$$Z - Z_S = -\frac{1}{\mu} \ln \left\{ \frac{\cos (\gamma_S + \mu x)}{\cos \gamma_S} \right\} \tag{22}$$

for any and all rays. This gives the limiting ray, which grazes the surface $Z = 0$ at angle $\gamma = 0$, to be

$$\gamma_{S,1} = \arccos e^{-\mu Z_S} \tag{23}$$

The maximum range for direct rays is thus

$$x_M = \frac{1}{\mu} \left(\frac{\pi}{2} - \arccos e^{-\mu Z_S}\right) \tag{24}$$

Equation (22) may be transformed into

$$\tan \gamma_S = \frac{\cos \mu x - e^{-\mu (Z - Z_S)}}{\sin \mu x} \tag{25}$$

which gives γ at any point within direct range. It is pertinent to note that the distribution is single valued. This means that no rays cross. There are no caustics -- a caustic being the locus of a wavefront folding point.

By definition

$$\nabla \gamma_S \equiv \text{m} \frac{1}{L} \tag{26}$$

where m is defined in Fig. 1 and L is referred to as the specific wavefront length per unit radian of emission angle. The ascendent operation on Eq. (25) yields

$$\sec^2 \gamma_S \ \nabla\gamma_S = \frac{\cos \mu x \ e^{-\mu (Z-Z_S)} - 1}{\sin^2 \mu x} \mu \nabla x$$

$$+ \frac{e^{-\mu (Z - Z_S)}}{\sin \mu x} \mu \nabla Z \tag{27}$$

where ∇x and ∇Z are orthogonal unit vectors. The magnitude, with sign consideration, leads to

$$L = \frac{1}{\mu} \left\{ 1 - 2 \cos \mu x \ e^{-\mu (Z-Z_S)} + e^{-2\mu (Z-Z_S)} \right\}^{1/2} \tag{28}$$

This is our analytic solution.

Equation (28) may also be transformed into

$$L = \frac{1}{\mu} \varphi^{1/2} e^{-\frac{\mu}{2}(Z - Z_S)} \tag{29}$$

where

$$\varphi \equiv 2 \cosh \mu (Z - Z_S) - 2 \cos \mu x \tag{30}$$

and which shows that L is positive everywhere.

For later usefulness we combine Eqs. (25) and (28) to form

$$L = \frac{\sin \mu x}{\mu \cos \gamma_S} \tag{31}$$

We also note that

$$\tan \gamma = \frac{e^{\mu (Z - Z_S)} - \cos \mu x}{\sin \mu x} \tag{32}$$

For an arbitrary wavefront in the x, Z plane,

$$-\frac{dx}{dZ} = \tan \gamma = \frac{e^{\mu (Z - Z_S)} - \cos \mu x}{\sin \mu x} \tag{33}$$

which integrates to yield the wavefront curve

$$2 \cos \mu x = e^{\mu(Z - Z_S)} + A\, e^{-\mu(Z - Z_S)} \tag{34}$$

The value of A determines the particular wavefront: For the wavefront passing through the point x_*, Z_S,

$$A = 2 \cos \mu x_* - 1 \tag{35}$$

3. Non-reciprocity by Analytical Solution

For an effective point source at arbitrary location, $x = 0$ and depth Z_S, Eq. (29) gives the specific wavefront length, L, at an arbitrary receiver location within range, at x_R and depth Z_R :

$$L_R = \frac{1}{\mu} \varphi_R^{1/2}\, e^{-\frac{\mu}{2}(Z_R - Z_S)} \tag{36}$$

where

$$\varphi_R \equiv 2 \cosh \mu\, (Z_R - Z_S) - 2 \cos \mu x_R \tag{37}$$

The particular direct ray, making the connection, is given by

$$\tan \gamma_S = \frac{\cos \mu x_R - e^{-\mu(Z_R - Z_S)}}{\sin \mu x_R} \tag{38}$$

If we now interchange locations of source and receiver then the specific wavefront length at Z_S, by ray traversing the same path as before but

in the reverse direction, is

$$L_S = \frac{1}{\mu} \varphi_S^{1/2} e^{-\frac{\mu}{2}(Z_S - Z_R)} \tag{39}$$

where

$$\varphi_S \equiv 2 \cosh \mu (Z_S - Z_R) - 2 \cos \mu x_R$$

$$= \varphi_R \tag{40}$$

We note that φ is an even function but the exponential factor in L is not. The ratio of the spreading amounts is

$$\frac{L_R}{L_S} = e^{-\mu(Z_R - Z_S)} = \frac{C_R}{C_S} \tag{41}$$

In our example the downward traverse of a path produces less spreading than the upward traverse of the same path.

We have shown by analytical example that the field of listening coverage for a receiver location cannot be obtained on the basis of equivalency by treating the receiver as a normalized source. However while not strictly equivalent such procedure may yield reasonable approximations under most conditions.

We can edify the non reciprocity in a more general way. For all ray paths along which

$$\frac{\partial^2 C}{\partial n^2} \equiv 0 \tag{42}$$

Eq. (6) reduces to

$$\frac{\partial^2 L}{\partial s^2} = \frac{\partial L}{\partial s} \frac{1}{C} \frac{\partial C}{\partial s} \tag{43}$$

Written as perfect differentials, this yields

$$\frac{\partial L}{\partial s} = \frac{C}{C_S} \tag{44}$$

where C_S is the sound speed at the source location. In integrating along such a ray path from a source at location A and a receiver at location B, we obtain

$$L_B = \frac{1}{C_A} \int_A^B C \, ds \tag{45}$$

Along the same path, for a source at location B and a receiver at location A, we obtain

$$L_A = \frac{1}{C_B} \int_A^B C \, ds \tag{46}$$

The ratio for alternate directions of traverse, yields

$$\frac{L_B}{L_A} = \frac{C_B}{C_A} \tag{47}$$

In this regard, intensity is maximized for a receiver located at a sound-speed minimum.

4. Analytical Test of Spreading Equation

Our analytic solution was completely derived from Eq. (1) with R a constant. This solution may be used as a test of other equations of Section 1 and any further derivations. We will use it to test Eq. (12) which reduces to Eq. (6) for $R = \infty$. We express Eq. (12) in the form

$$\frac{\partial^2 L}{\partial s^2} = -\frac{L}{C}\left\{\frac{\partial^2 C}{\partial n^2} - \frac{1}{L}\frac{\partial C}{\partial s}\frac{\partial L}{\partial s}\right\}$$

$$- L\left\{\frac{2\cos\gamma}{R\ C}\frac{\partial C}{\partial n} + \frac{\cos^2\gamma}{R^2} - \frac{\sin\gamma}{R\,L}\frac{\partial L}{\partial s}\right\} \qquad (48)$$

The test we make is with L in the form expressed by Eq. (31). Noting that

$$\frac{\partial \gamma_S}{\partial s} = 0 \qquad (49)$$

$$\frac{\partial \gamma}{\partial s} = \mu\frac{\partial x}{\partial s} = \mu\cos\gamma \qquad (50)$$

we obtain

$$\frac{\partial L}{\partial s} = \frac{\cos\mu x\cos\gamma}{\cos\gamma_S} = \mu L\frac{\cos\gamma}{\tan\mu x} \qquad (51)$$

$$\frac{\partial^2 L}{\partial s^2} = -L \left\{ \mu^2 \cos^2 \gamma + \mu^2 \frac{\sin \gamma \cos \gamma}{\tan \mu x} \right\} \tag{52}$$

$$\frac{\partial C}{\partial n} = -k C \cos \gamma \tag{53}$$

$$\frac{\partial^2 C}{\partial n^2} = +k^2 C \cos^2 \gamma \tag{54}$$

$$\frac{\partial C}{\partial s} = -k C \sin \gamma \tag{55}$$

Substitution of Eqs. (51) through (55) in Eq. (48), with $\mu \equiv k - 1/R$, readily shows identity.

5. A Test of the Computer Programs

In applying the analytic solution of Section 2 as a test for the computer programs of the Fleet Numerical Weather Facility we must specify numerical values. For the sound-speed profile, Eq. (16), we specify

$$C_0 = 1{,}680 \text{ meters seconds}^{-1}$$

$$k = 2 \times 10^{-5} \text{ meters}^{-1}$$

and submit the sound-speed profile values:

Z (meters)	C (meters seconds^{-1})
0	1,680.00
500	1,663.28
1000	1,646.73
1500	1,630.35
2000	1,614.13
2500	1,598.06
3000	1,582.17
3500	1,566.42
4000	1,550.83
4500	1,535.40
5000	1,520.13
5500	1,505.00
6000	1,490.03
6500	1,475.20
7000	1,460.52
7500	1,445.99
8000	1,431.60
8500	1,417.36

Tests one and two were run without the curvature factor. That is, $R = \infty$, and the rays are obtained by integrating Eq. (3) and the spreading factor by Eq. (6).

In Test one the source is specified at the 1000 meter depth. The bottom is specified at 8,500 meters. The rays, γ_S, from -10° to $+10^\circ$ in two-degree intervals, are requested. The integration of each ray is terminated at the first bottom intercept. From the output we select the results shown in Table I; we also show evaluations of Eqs. (25) and (28) for the <u>same locations</u>.

Table I: No Curvature

Eqs. (3) and (6) Integrated				Eqs. (25) and (28) Evaluated	
x (yds)	Z (feet)	γ_S (degrees)	L (yds)	γ_S (degrees)	L (yds)
40,109	27,887.14	- 10.00	37,220.0	- 9.99	$37{,}17_0$
35,437	27,887.14	- 6.00	33,175.4	- 6.00	$33{,}19_0$
31,174	27,887.14	- 2.00	29,528.9	- 2.00	$29{,}53_1$
29,209	27,887.14	0.00	27,841.3	0.00	$27{,}84_0$
23,984	27,887.14	6.00	23,352.4	6.00	$23{,}35_6$
21,039	27,887.14	10.00	20,841.3	10.00	$20{,}84_0$
1,909	3,180.87	- 2.00	1,909.7	- 2.00	$1{,}91_0$

Test two has the source at the bottom, 8,500 meters, and selects rays, γ_S, as the reflected bottom intercept angles of Test one. We show only a result for the reversal of the zero ray of Test one for which we evaluate L by Eq. (31) for the given γ_S :

x (yds)	Z (feet)	γ_S (degrees)	L (yds)	Eq. (31)
29,209	3,280.84	- 30.60	32,362.9	32,344

In the opposite propagation the spreading for this ray path, given by Test one, amounted to 27,841 yards.

Test three repeats Test one but with curvature; R = 6,371 km, making $\mu = 1.9843 \times 10^{-5}$. The numerical program obtains the rays by integration of Eq. (1) and the spreading by integration of Eq. (6). Our analytical solution, however, satisfies Eq. (12). This inconsistency in the treatment of curvature is included with truncation errors in the disparity between the integrated results and the evaluation of the

analytic solutions. The selected results are shown in Table II.

Table II: With Curvature

Eqs. (1) and (6) Integrated				Eqs. (25) then (31) Evaluated	
x (yds)	Z (feet)	γ_S (degrees)	L (yds)	γ_S (degrees)	L (yds)
40,323	27,887.14	- 10.00	37,401.2	- 9.99	$37,38_7$
35,609	27,887.14	- 6.00	33,304.0	- 6.00	$33,36_5$
31,311	27,887.14	- 2.00	29,633.7	- 2.00	$29,67_2$
29,330	27,887.14	0.00	27,934.9	0.00	$27,96_5$
24,066	27,887.14	6.00	23,416.7	6.00	$23,43_6$
21,102	27,887.14	10.00	20,890.2	10.00	$20,90_8$
1,924	3,180.08	- 2.00	1,924.8	- 2.00	$1,92_5$

These results check out the particular FNWF computer programs and attest to their accuracy.

6. A Note on the Lateral Spreading

In general a wavefront is a surface having two dimensions of spreading. The specific wavefront area may be expressed by

$$E = H L \tag{56}$$

where H is the specific wavefront length measured horizontally and L is the specific wavefront length measured normal to H. Both H and L may be integrated along a ray according to

$$\frac{\partial^2 H}{\partial s^2} = -\frac{H}{C}\left\{\frac{\partial^2 C}{\partial h^2} - \frac{1}{H}\frac{\partial C}{\partial s}\frac{\partial H}{\partial s}\right\} \tag{57}$$

$$\frac{\partial^2 L}{\partial s^2} = -\frac{L}{C}\left\{\frac{\partial^2 C}{\partial n^2} - \frac{1}{L}\frac{\partial C}{\partial s}\frac{\partial L}{\partial s}\right\} \tag{58}$$

where h is a linear coordinate along the horizontal normal to the ray and n is normal to coordinates h and s.

Sections 1 through 5 deal with ray tracing in a vertical x, Z plane and with the L component of the spreading. That the rays stay in the plane implies the condition, or approximation, that

$$\frac{\partial C}{\partial y} \equiv 0 \tag{59}$$

where y is normal to the x, Z plane. If we further assume that

$$\frac{\partial^2 C}{\partial h^2} \equiv \frac{\partial^2 C}{\partial y^2} \equiv 0 \tag{60}$$

then Eq. (57) reduces to

$$\frac{\partial^2 H}{\partial s^2} = \frac{\partial H}{\partial s}\frac{1}{C}\frac{\partial C}{\partial s} \tag{61}$$

Eq. (61) is a perfect-differential relationship which yields

$$\frac{\partial H}{\partial s} = \frac{C}{C_S} \tag{62}$$

where C_S is the speed of sound at the source location. The integration of H takes the form

$$\boxed{H = \frac{1}{C_S} \int_0^s C \, ds} \tag{63}$$

Again we note that there is no reciprocity in interchanging source and receiver locations for a specific ray path. The ratio of spreading amounts in the two directions is inverse to the ratio of source speeds, again implying best listening at speed-of-sound minima.

For speed-of-sound as a function of depth only, we have Snell's law:

$$\frac{\cos \gamma}{\cos \gamma_S} = \frac{C}{C_S} \tag{64}$$

Substitution in Eq. (63) yields

$$H = \frac{1}{\cos \gamma_S} \int_0^s \cos \gamma \, ds = \frac{x}{\cos \gamma_S} \tag{65}$$

A generalization of the last expression may be obtained for radial symmetry in the speed-of-sound distribution; the symmetry is that expressed by $x \equiv R$, the <u>horizontal</u> range in all directions from the source location.